Engineering Notes

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Remarks on Conical Supersonic Wings with Subsonic Leading Edges

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SUPERSONIC conical wings have been treated by many authors, since conical flowfields can be calculated relatively simple using linearized theory. In Ref. 1 a formula is given directly relating the pressure difference $\Delta C_p = C_{p_l} - C_{p_u}$ to the angle of attack distribution $\alpha(\eta) = -w/V_\infty$ of such wings with subsonic leading edges. Using the formula of Ref. 1, a family of conically twisted wings has been developed, by Holla and others,[2] for which all integrations can be carried out analytically. Now a further interesting case will be added to this family, namely

$$\mathrm{d}(w/V_\infty)/\mathrm{d}\bar{\eta} = c\bar{\eta}/\sqrt{1-\bar{\eta}^2} \tag{1}$$

where c is an arbitrary constant and $\bar{\eta} = y/(x\ \tan\mu)$ is a dimensionless spanwise coordinate (μ = Mach angle, x = streamwise coordinate). Introduced in the basic formula, this leads after integration to

$$\frac{\Delta C_p}{\tan\omega} = \frac{4}{\sqrt{m^2-\bar{\eta}^2}}\,\frac{m}{E'(m)} \times \{\alpha + c[E'(m) - 1 - (1-\sqrt{1-m^2})K'(m)]\} + \frac{4}{m}c\left(\arctan\sqrt{\frac{m^2-\bar{\eta}^2}{1-m^2}} - \sqrt{m^2-\bar{\eta}^2}\right) \tag{2}$$

with $m = \tan\omega/\tan\mu$. $K'(m)$ and $E'(m)$ are the complete elliptic integrals of the first and second kind respectively, with the complementary modulus $\sqrt{1-m^2}$. $\alpha = \alpha(0)$ is the angle of attack in the plane of symmetry of the wing (x,z plane). The surface shape can be expressed by

$$\zeta = \frac{z}{x} = \nu\ \tan\omega\eta - \eta\int_1^\eta \frac{\alpha + w/V_\infty}{\eta'^2}\,\mathrm{d}\eta' \tag{3}$$

where ν is a small arbitrary dihedral angle and $\eta = y/(x\ \tan\omega)$. For the case of surfaces without a slope discontinuity at $\eta = 0$, one finds from Eqs. (1) and (3) that

$$\zeta = c(1 - \sqrt{1-\bar{\eta}^2} - \bar{\eta}\ \arcsin\bar{\eta}) \tag{4}$$

and

$$c = f/(\sqrt{1-m^2} + m\ \arcsin m - 1) \tag{4a}$$

with f being the camber value $\zeta(0) - \zeta(\bar{\eta} = m)$. Equation (4) implies only small changes in the shape from $m = 0$ to $m = 1$ but more considerable deviations in local angle of attack. Therefore the results for the pressure distributions of this [Eq. (2)] and the other cases[2] do not depend solely on aerodynamic effects with changing m but also on shape modifications.

To separate both these influences, in addition the case of fixed parabolic twist ($\zeta = -f\eta^2$) has been treated, which is the limiting case of Eq. (4) for $m \to 0$. The corresponding pressure distribution for $m = 0$ can be derived analytically from Eq. (2) by a series expansion for $m \to 0$ or by slender-body theory.[3] For all other values of m the basic formula must be partially evaluated numerically.

Pressure distributions without leading-edge singularity for shapes after Eq. (4) and for parabolic twist are shown in Fig. 1. Up to medium values of m only small changes occur in the pressure distributions. They grow up significantly if m approaches 1, in particular for the shapes after Eq. (4). That must be primarily a consequence of changes in the angle of attack for vanishing leading-edge singularity, since the ΔC_p values for a triangular flat plate decrease with m. Further, from the parabolic twist results for $m = 0$ and $m = 1$ the outward shift of maximum ΔC_p with growing m can be seen as an essential aerodynamic effect for cambered wings, and not due to geometric changes only, as could be supposed from results for the shape family after Ref. 2 and Eq. (1). For the shapes after Eq. (4) this maximum is even shifted into the leading edge because of an infinite shape curvature at this edge for $m = 1$. (The contours of Ref. 2 exhibit zero curvature in this case, but these features are not of practical importance since nonlinear transonic effects are dominant if the leading edge becomes a sonic edge.)

Moreover, when an exact solution for conical twist is known, it is of some interest to check the usual approximations associated with Evvard's[4] theorem. Terminating Hancock's series expansion[5] after the first term, the largest deviations from the exact solution occur for $m \to 0$. Therefore in Fig. 2, where the appropriate integration regions are indicated by hatching, this first approximation is compared with the exact results for small m ($m = 0.1$), for which the pressure distributions belonging to the shapes after Eq. (4) and parabolic twist do not differ significantly from each other. The first approximation exceeds the exact solution by only 8%, in contrast to the case of a flat triangular plate where the corresponding difference is about 18%. Further, since the angle of attack for a vanishing leading-edge

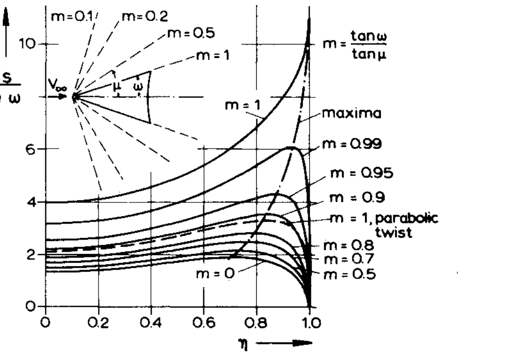

Fig. 1 Pressure distributions for vanishing leading-edge singularity for shapes after Eq. (4) and for parabolic twist.

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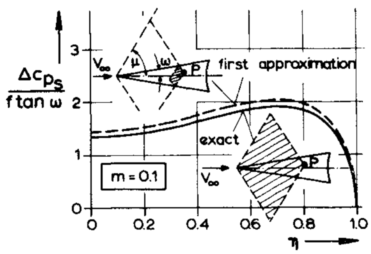

Fig. 2 Comparison between exact pressure distribution and first approximation (integration regions hatched).

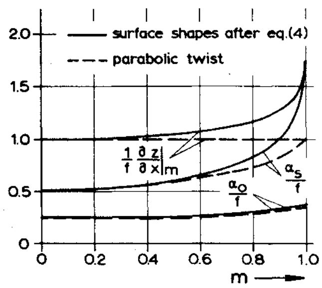

Fig. 3 Angles of attack for vanishing leading-edge singularity and for zero lift.

singularity does not change noticeably (see below), the first approximation for a cambered wing is better than for a flat one.

The case of zero (or finite) leading-edge pressure can be obtained from Eq. (2) by requiring the singular part to vanish. From this, the proper angle of attack is obtained for shapes after Eq. (4) as

$$\alpha_s/f = c/f[1+(1-\sqrt{1-m^2})K'(m)-E'(m)] \tag{5}$$

Also, for parabolic twist this angle can be expressed analytically as

$$\alpha_s/f = 2/\pi K(m)K'(m) + 2/m^2\{1-1/\pi E'(m)[E(m)+K(m)]\} \tag{6}$$

with $K(m)$ and $E(m)$ being the complete elliptic integrals of the first and second kind respectively. The angle of attack for zero lift follows from Eq. (2) by integrating the nonsingular part to find C_{L_s} and using the well-known result for the lift curve slope[1]

$$\alpha_0/f = \alpha_s/f - C_{L_s}/(fC_{L_\alpha}) = c/f\{1-E'(m)/2 + (1-\sqrt{1-m^2})[K'(m)-E'(m)/m^2]\} \tag{7}$$

For parabolic twist this characteristic angle has to be evaluated numerically, except for the limiting cases:

for $m=0$ $\alpha_0/f = 1/4$

for $m=1$ $\alpha_0/f = 1/3$

In Fig. 3 the characteristic angles of attack are shown and the streamwise contour slopes at the leading edge are also plotted for comparison. The typical increase of α_s (vanishing leading-edge singularity) for $m \rightarrow 1$ can be seen. In the case of parabolic twist this is due solely to the diminishing streamline slope in front of the leading edge and remains remarkably lower than that in the case of Eq. (4). On the other hand, the zero-lift angles of attack differ only by a small amount. Furthermore, all differences between the exact solution and the first approximation discussed above are too small to be plotted in this figure.

More details of the analysis can be found in Ref. 6.

References

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[4]Evvard, J. C., "Effects of Yawing Thin Pointed Wings at Supersonic Speeds," NACA TN 1429, 1947.

[5]Hancock, G. J., "Note on the Extension of Evvard's Method to Wings with Subsonic Leading Edges Moving at Supersonic Speeds," *Aeronautical Quarterly*, Vol. VIII, 1957, pp. 87-102.

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Errata

Remarks on Thin Airfoil Theory

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IN the above titled Note, the general solution for the integral

$$J_n(\phi) \equiv \int_0^\pi \frac{\sin n\theta}{\cos\theta - \cos\phi} d\theta \tag{1}$$

is found to be in error. The integral has the recurrence relation

$$J_{n+1} + J_{n-1} = 2\cos\phi J_n + (2/n)[1-(-1)^n] \tag{2}$$

with starting values

$$J_0 = 0 \tag{3}$$

$$J_1 = 2 \log \tan(\phi/2) \tag{4}$$

The subsequent derivation in Ref. 1 for finding a general solution for Eq. (2) appears to be in error. The correct solution is determined to be (for $n \geq 1$)

$$J_n = 2 \log \tan\frac{\phi}{2} \frac{\sin n\phi}{\sin\phi} + \frac{2}{\sin\phi} \sum_{k=1}^{n-1} \frac{[1-(-1)^k]}{k} \sin(n-k)\phi \tag{5}$$

and may be verified by substitution in Eq. (2). The final solution, Eq. (5), was written down by inspection after obtaining J_0 to J_{10} in explicit form using Eqs. (2-4).

For computational efficiency it is preferable to use Eqs. (2-4), since it avoids a large number of trigonometric function evaluations that would otherwise be required if Eq. (5) is used.

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